Optimal Resilient Distribution Grid Design
Extended Abstract

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1 Introduction

Natural disasters such as earthquakes, hurricanes, and other extreme weather pose serious risks to modern critical infrastructure including electrical distribution grids. Recent U.S. government sources [1, 2] suggest that new methodologies for improving system resilience to these events is necessary. Here, we focus on developing methods for designing and upgrading these grids to better withstand and recover from these threats. These tools select from a set of potential distribution grid upgrades proposed by planners, e.g. adding lines, transformers, breakers, and microgrids (modeled as distributed generation), so that the post-event damaged grid meets a minimum standard of service (98% of the critical demand is met and 50% of all other load is served) with minimum upgrade budget. The approach currently addresses events that create relatively homogeneous damage, e.g. ice storms. When the damage is known apriori, a mixed integer programming formulation is tractable, and we present some initial results.

2 Model

The distribution grid is modeled as a three-phase system with a set of nodes $\mathcal{N}$ and edges (lines and transformers) $\mathcal{E}$. Each node, i, is associated with the following design constants: $\mathcal{P}_i$, the set of phases allowed to consume or inject; d_i^k, the demand for power on phase k; Z_i^k, the maximum amount of microgrid capacity on phase k; μ_i, the fixed cost to build microgrid generation; and ζ_i^k, the per MW cost to build microgrid generation on phase k. Each node, i, is also associated with the following decision variables: u_i, a binary variable for building a microgrid; z_i^k, the real power capacity for microgrid generation on phase k; g_i^k, the real power generated on phase k; l_i^k, the real power delivered on phase k; and y_i, a binary variable for determining if (all) the load at i is served or not.

An undirected edge e is indexed by i, j. Ordering of the indexes indicates the positive direction for variables such as power flow. Each edge i, j is associated with the following design constants: $\mathcal{P}_{i,j}$, the set of phases allowed on the line; $c_{i,j}$, the cost to build a new line; $\kappa_{i,j}$, the cost to build a switch; and $Q_{i,j}^k$, the real power capacity on phase k. Each edge i, j is also associated with the following decision variables: $x_{i,j}$ is a binary variable for determining if the line is built; $\tau_{i,j}$, a binary variable for building a switch; and $f_{i,j}^k$ which denotes the flow on phase k. Finally, we use the notation S to denote the set of all possible loops in the grid, λ to denote how much critical load must be served, γ to denote how much of non-critical load must be served, and $\mathcal{L} \in \mathcal{N}$ denotes set of buses whose load is critical.

Figure 1 describes the optimization model. Equation 1 minimizes the cost of building lines, installing new switches and adding generation. The cost of single and three phase underground lines is between $\$40k$ and $\$1500k$ per mile [3] and we adopt the cost of $\$100k$ per mile and $\$500k$

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\begin{aligned}
\text{minimize} \quad & \sum_{i,j\in\mathcal{E}} c_{i,j}x_{i,j} + \sum_{i,j\in\mathcal{E}} \kappa_{i,j}\tau_{i,j} + \sum_{i\in\mathcal{N},k\in\mathcal{P}_i} \zeta_i^k z_i^k + \sum_{i\in\mathcal{N}} \mu_i u_i && && (1)\\
\text{subject to} \quad & -x_{i,j}Q_{i,j}^k|\mathcal{P}_{i,j}| \leq \sum_{k\in\mathcal{P}_{i,j}} f_{i,j}^k \leq x_{i,j}Q_{i,j}^k|\mathcal{P}_{i,j}| && \forall i,j\in\mathcal{E}, k\in\mathcal{P}_{i,j} && (2)\\
& -(1-\tau_{i,j})Q_{i,j}^k|\mathcal{P}_{i,j}| \leq \sum_{k\in\mathcal{P}_{i,j}} f_{i,j}^k \leq (1-\tau_{i,j})Q_{i,j}^k|\mathcal{P}_{i,j}| && \forall i,j\in\mathcal{E}, k\in\mathcal{P}_{i,j} && (3)\\
& -\beta_{i,j}\frac{\sum_{k\in\mathcal{P}_{i,j}} f_{ij}^k}{|\mathcal{P}_{i,j}|} \leq f_{i,j}^{k'} - \frac{\sum_{k\in\mathcal{P}_{i,j}} f_{ij}^k}{|\mathcal{P}_{i,j}|} \leq \ \beta_{i,j}\frac{\sum_{k\in\mathcal{P}_{i,j}} f_{ij}^k}{|\mathcal{P}_{i,j}|} && \forall i,j\in\mathcal{E}\forall k'\in\mathcal{P}_{i,j} && (4)\\
& l_i^k = y_i d_i^k && \forall i\in\mathcal{N}\forall k\in\mathcal{P}_i && (5)\\
& 0 \leq g_i^k \leq z_i^k && \forall i\in\mathcal{N}\forall k\in\mathcal{P}_i && (6)\\
& g_i^k - l_i^k - \sum_{j\in\mathcal{N}} f_{i,j}^k = 0 && \forall i\in\mathcal{N}\forall k\in\mathcal{P}_i && (7)\\
& 0 \leq z_i^k \leq \ u_i Z_i^k && \forall i\in\mathcal{N}, k\in\mathcal{P}_i && (8)\\
& \sum_{i,j\in s}(x_{i,j} + (1-\tau_{i,j})) \leq |s| - 1 && \forall s\in S && (9)\\
& \sum_{i\in\mathcal{L},k\in\mathcal{P}_i} l_i^k \geq \lambda \sum_{i\in\mathcal{L},k\in\mathcal{P}_i} d_i^k && && (10)\\
& \sum_{i\in\mathcal{N}\setminus\mathcal{L},k\in\mathcal{P}_i} l_i^k \geq \gamma \sum_{i\in\mathcal{N}\setminus\mathcal{L},k\in\mathcal{P}_i} d_i^k && && (11)\\
& x, y, \tau, u \in \{0,1\} && && (12)
\end{aligned}
$$

Figure 1: Optimal Resilience for Distribution Grids.

per mile, respectively. The cost of single and three phase switches is estimated to be \$10k and \$15k, respectively [4]. Finally, the installed cost of natural gas-fired CHP in a microgrid is estimated to be \$1500k per MW [5].

Equations 2 and 3 limit the average flow at all phases to less than their thermal capacity. When the line is not built or is switched open, the flow is forced to zero. Equation 4 limits the phase imbalance to less than $\beta = 0.15$ which is a typical criterion for long-term operations [6]. Equation 5 restricts load shedding to a binary decision, i.e. no load is shed or all the load is shed at a node. Equation 6 limits generation at a node to less than the microgrid capacity. Equation 7 ensures the balance of all phase currents at all nodes. Equation 8 limits the amount of new generation installed in a microgrid at a node. Equation 9 states that the switches must be operated such that the resulting circuits are trees, i.e. with no loops. The systems considered here have a small number of possible loops. In general, there are an exponential number of loops in graphs, and alternate ways exist to handle this constraint for a large number of loops. With $\lambda = 0.98$, Eq. 10 ensures that the required 98% of critical load is served. With $\gamma = 0.5$, Equation 11 ensures that the required 50% of total load is served. Finally, Equation 12 states which variables are discrete.

3 Empirical Results

To test our optimal design and operations model, we constructed a prototypical distribution grid by replicating the IEEE 34-Bus distribution test system [7] three times. The nodes of three circuits were laid out geographically using two different scalings to represent Urban and Residential settings. The potential upgrades included: sixteen new undergound lines (within and interconnecting the three circuits), five new switches, and all nodes were potential microgrid sites. The base model is then damaged in a homogeneous manner using a fragility model corresponding to wooden distribution pole breakage rates due to ice build up and wind loading on the wires spanning the poles [8]. The individually damaged networks are then upgraded using our optimization model to meet the performance requirements in Eqs. 10 and 11.

Figure 2 presents the results. Each dot in the graph represents the average of the minimal upgrade budget for 100 samples of each damage intensity. Here, "intensity" is measured in terms of the expected number of pole failures per circuit mile. The expected upgrade budget for Urban networks is less than Residential because the cost of building underground interconnecting lines is less due to shorter distances between nodes. Indeed, the solutions to the Urban cases are dominated by the building of redundant lines that keep the three systems fully connected. In contrast, the solutions to the Residential cases are dominated by the use of microgrids to serve very local islands. In all cases, the time required to solve these models was less than 0.1 CPU second on a typical laptop.

4 Conclusion

The preliminary work described here introduced a mixed-integer programming formulation for designing resilient distribution grids with a focus on single events with known damage. By sampling over many statistically similar events, we have discovered several qualitative features of resilient distribution grids in different settings. In future work, we will consider stochastic and robust versions of this problem where the event is unknown, but damage continues to be governed by an underlying probability model. We will expand the networks we consider to explore the computational tractibility of our approach. Future work will incorporate reactive power flows and voltage magnitudes, which are ignored in the current work.

References

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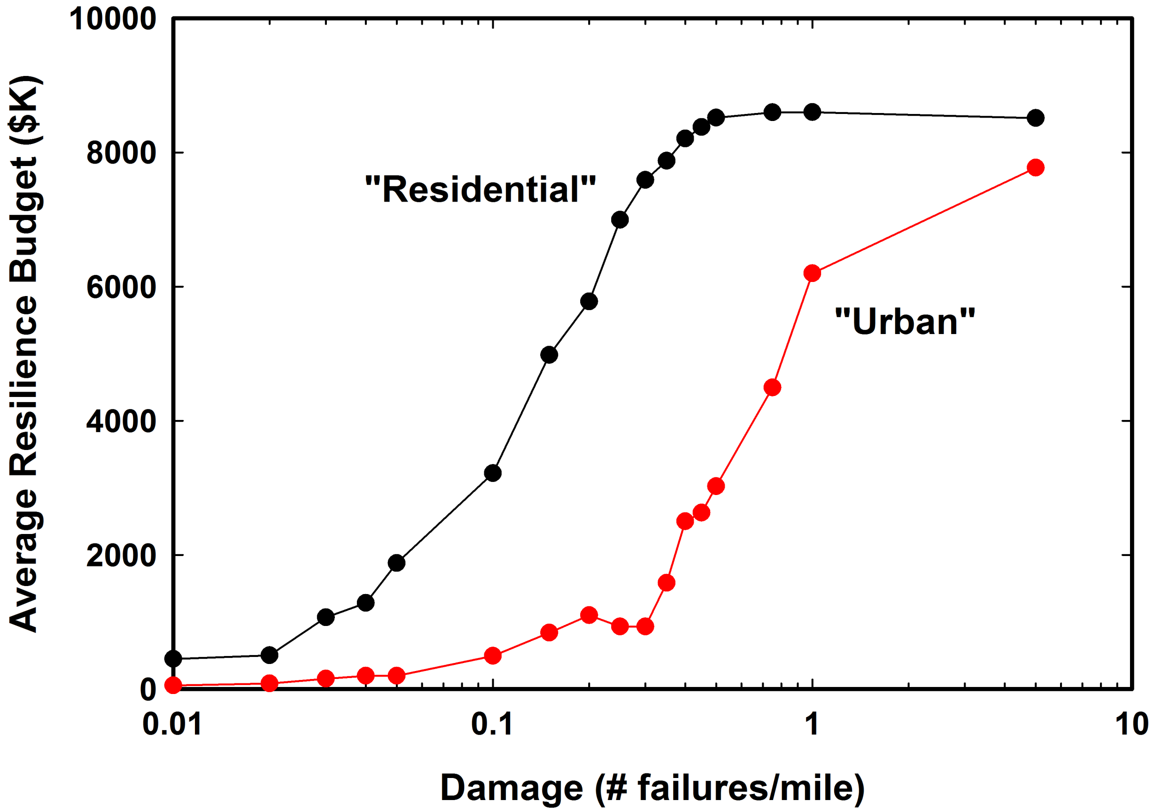

Figure 2: Average of minimal upgrade budgets for 100 independent samples of damage to a prototypical distribution grid composed of three replicas of the IEEE 34-Bus test circuit [7]. The Urban and Residential cases correspond to different geographical scalings of the prototypical grid to emulate the expected spatial extent of grids in these two environments.

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