Applications of Nonlinear Dynamics to Resiliency Analysis

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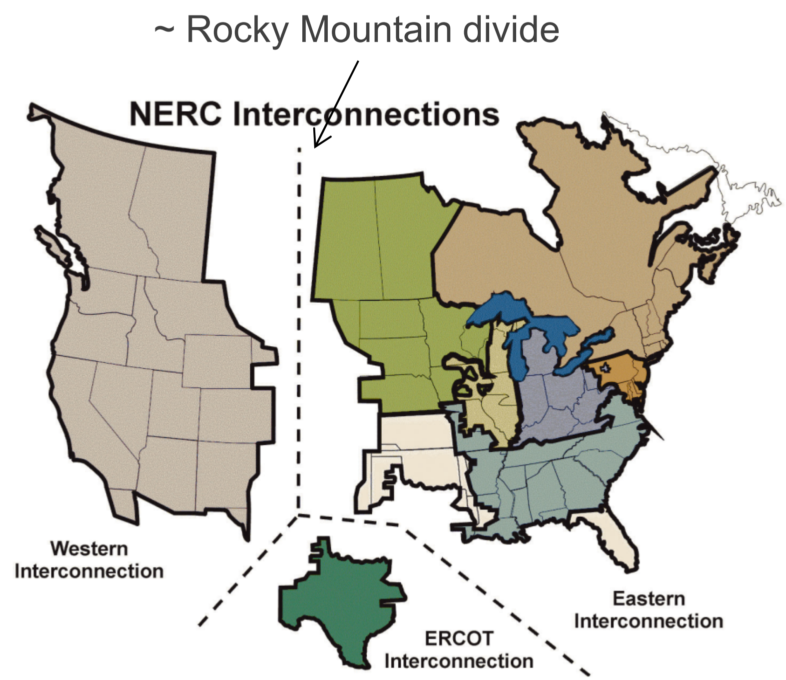
Resiliency of Critical infrastructure (CI) – Electrical power grid (EPG) as an example

- Resiliency is defined as "the ability of an entity — e.g., asset, organization, community, region — to anticipate, resist, absorb, respond to, adapt to, and recover from a disturbance from either natural or man-made events."
- Important category of such entities - CI.
- Energy, environmental, and critical materials domains
- Example - Electrical power grid



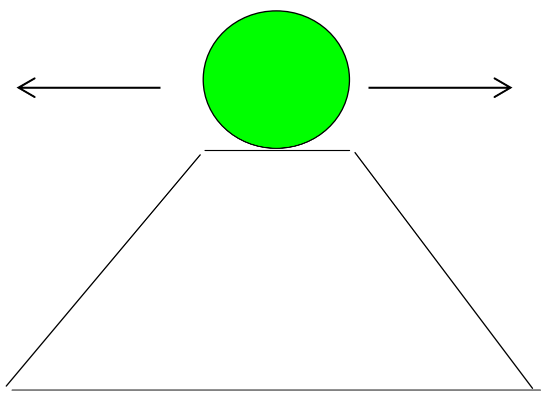
The North American EPG*

*Adapted from "National Transmission Grid Study," DOE 2002

Nonlinear dynamics (NLD) is a rigorous, general, mathematical tool for analyzing the resiliency of CI

- NLD can be used to manage a CI to anticipate, resist, absorb, respond to, adapt to, and recover from a disturbance from either natural or man-made events.
- Many large-scale, man-made or natural systems are *nonlinear* in their spatiotemporal behavior.
- Nonlinear - the response of the system to external perturbations is *stronger* than being proportional to the magnitude of the perturbations.
- May lead to *runaway*, loss-of-equilibrium situations.
- Example - brownout, blackout for EPG

Ball though in “equilibrium” is unstable against perturbation displacement in arrow’s direction

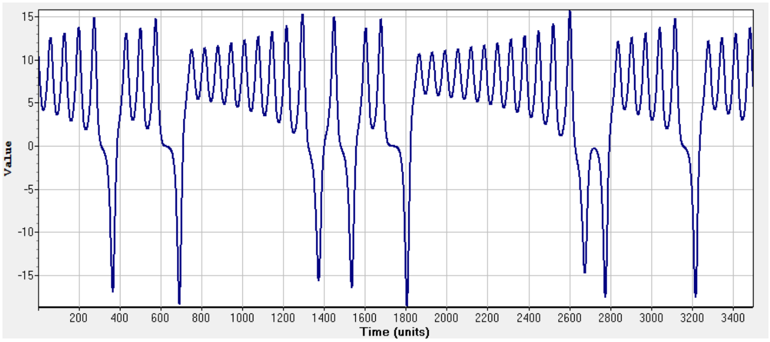


Many large-scale, man-made or natural systems are *nonlinear* in their spatiotemporal behavior.

How/Why NLD is useful

- The behavior of a CI over time may be governed by deterministic rules.
- May depend on ***many variables***.
- A plot of the evolutionary trajectory of a CI over all the relevant variables may show a ***multi-dimensional*** geometrical structure called an "attractor."
- In practice, one may be able to empirically monitor ***only one*** of the variables over time. Can one reconstruct the entire multi-dimensional CI (attractor) behavior from a single variable time series alone?
- Example: the Lorentz attractor

- Yes! From "one-dimensional" time series to multi-dimensional attractor



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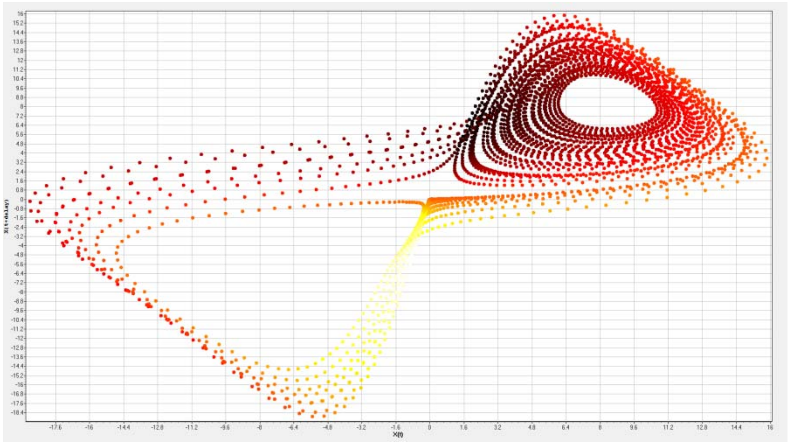
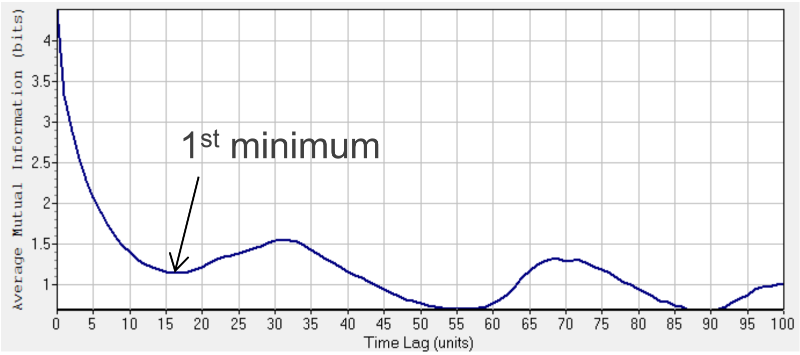


It is possible to reconstruct the full multi-dimensional behavior of a CI from a single variable time series of the same CI.

Reconstructing phase space attractor in higher dimensions

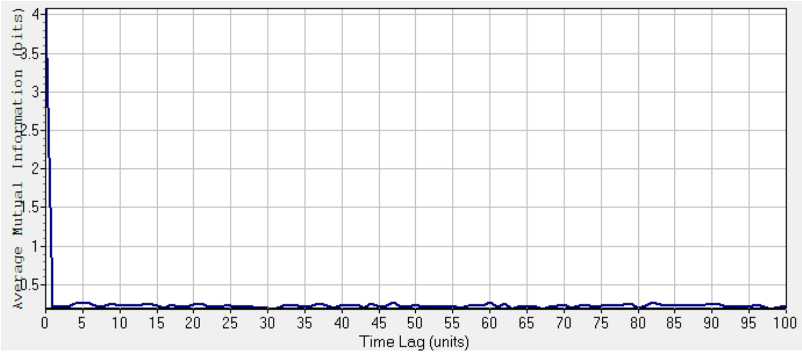
- Start with the “original” 1-dimensional time series related to the temporal behavior of a CI, $\{x_1, x_2, \dots x_{1+\delta}, \dots x_n\}$
- Choose a time delay step δ.
- Reconstruct a “2-dimensional” time series (phase space attractor)…
- $\{(x_1, x_{1+\delta}), (x_2, x_{2+\delta}), \dots, (x_{n-\delta}, x_n)\}$
- Such a process is called embedding into 2 dimensions, ….
- The “best” time delay step δ is chosen from the first minimum of Mutual Information (MI) function

MI for Lorentz attractor vs time delay δ



Deterministic

MI for random signal vs time delay δ

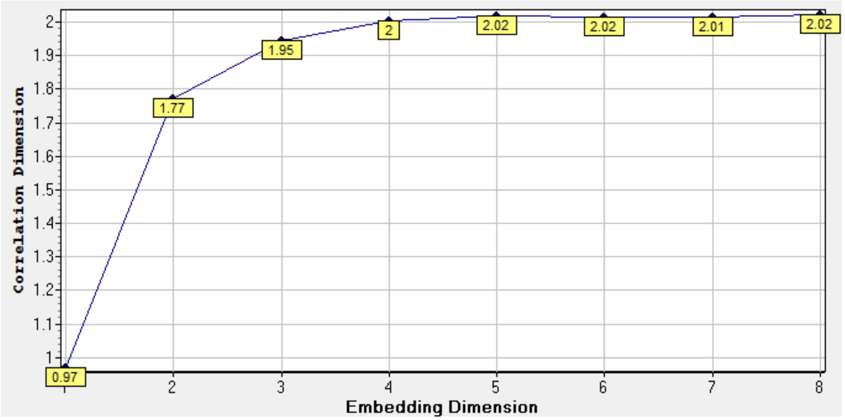


Noise

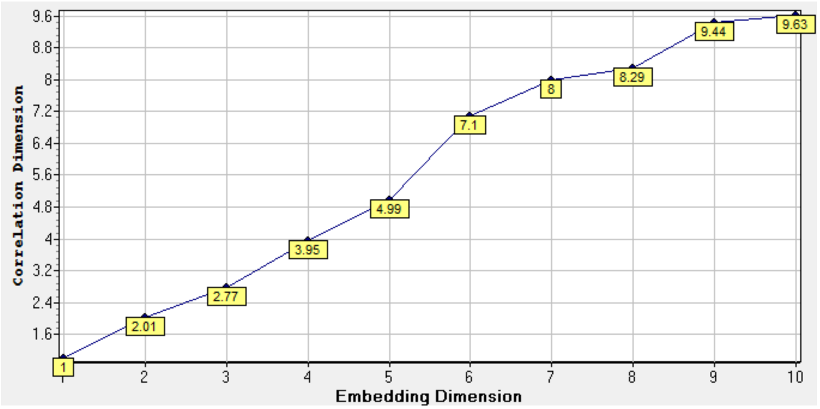
Extracting characteristics of the dynamical behavior of the system

- What is the maximum degree of freedom of the system?
- Information extractable from NLD analysis on time series data based on only a single degree of freedom!
- The correlation dimension D is an NLD construct that characterizes the degree of freedom of the system.
- D ~ 2.03, a non-integer?

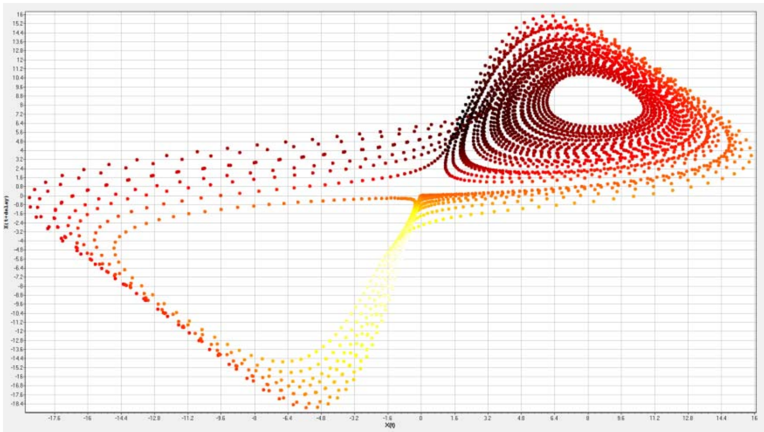
D vs embedding dimension for Lorentz



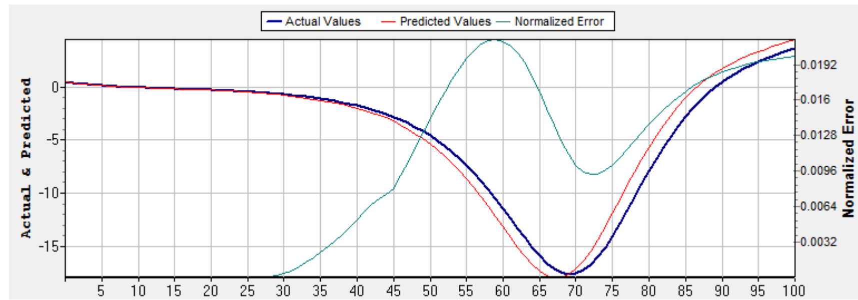
D vs embedding dimension for Noise



Short–term predictability of complex system evolution by NLD – Using a Lorentz system as an example

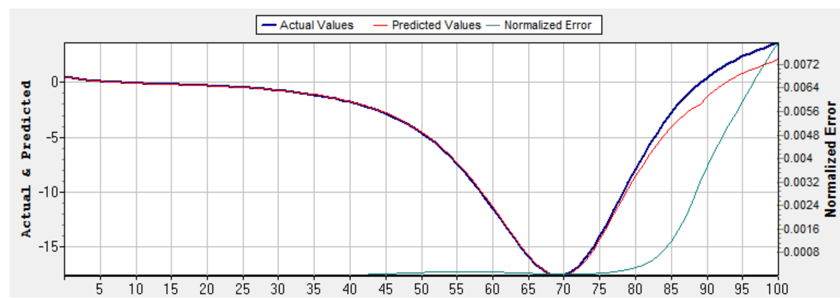


Embedding in 2 dimensions



- Attractor looks like it has a protrusion into the 3rd dimension.
- Knowing the “correct” D is important.
- Affects short-term predictability.

Embedding in 3 dimensions

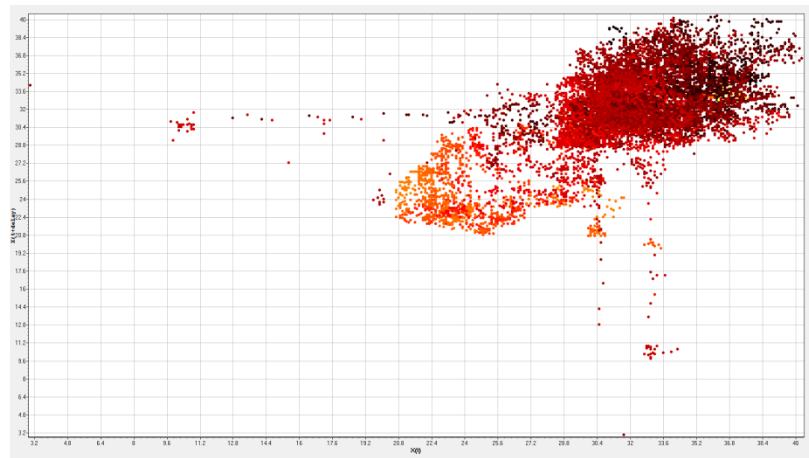


Embedding in 3 dimensions improves short-term predictability.

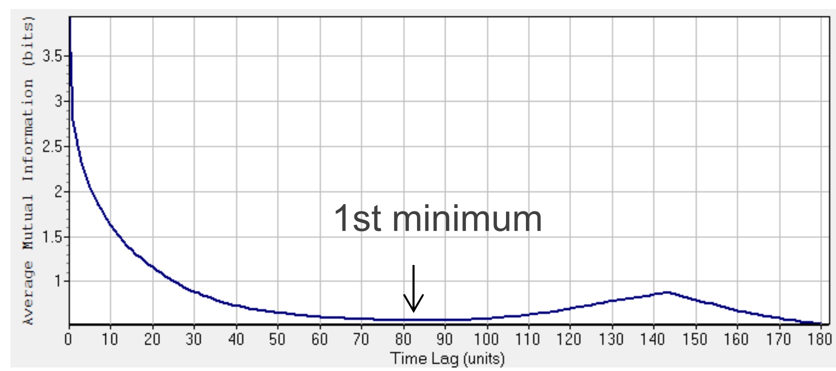
Applying NLD to analyze CI resiliency - The Argonne electrical grid as an example

Example based on actual Argonne electrical usage from the summer of 1999. The data provided courtesy of the local electrical utility.

Phase space representation of the Argonne electrical grid



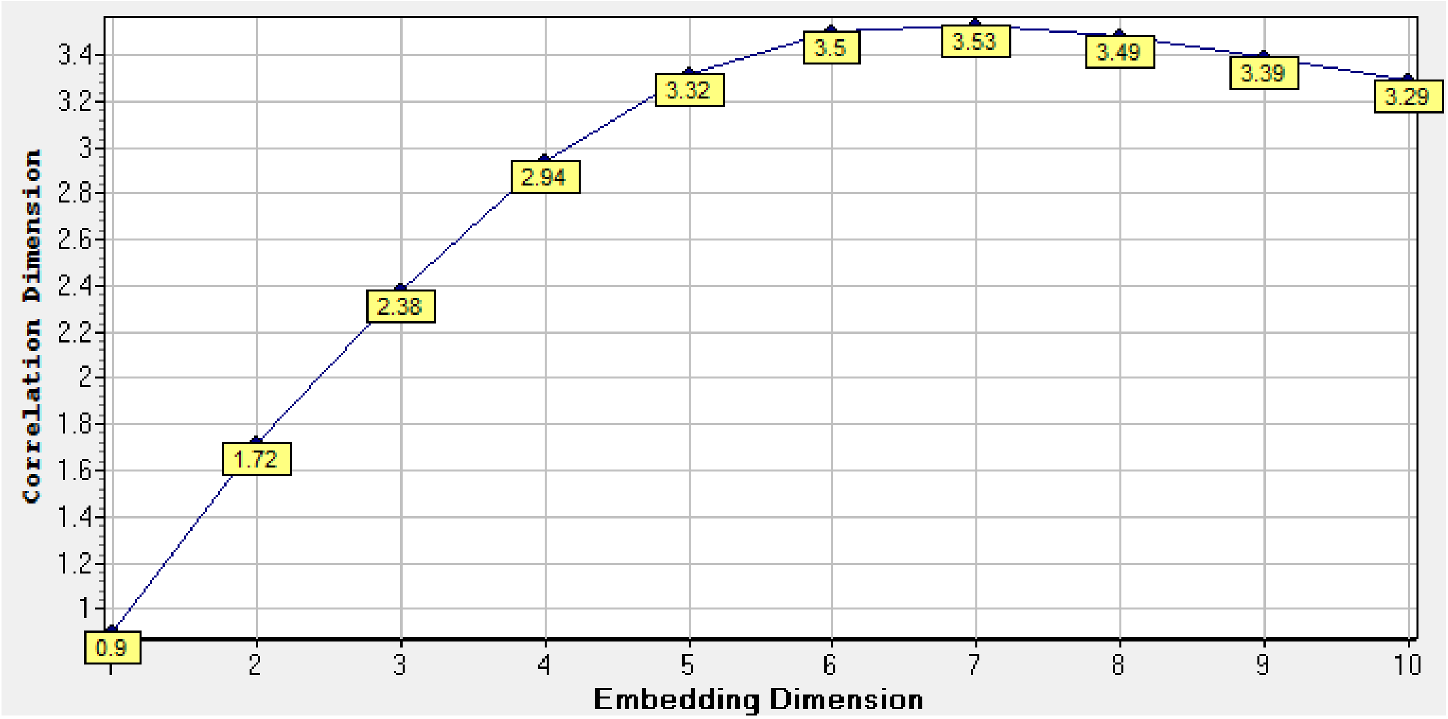
MI for Argonne electrical grid vs time delay



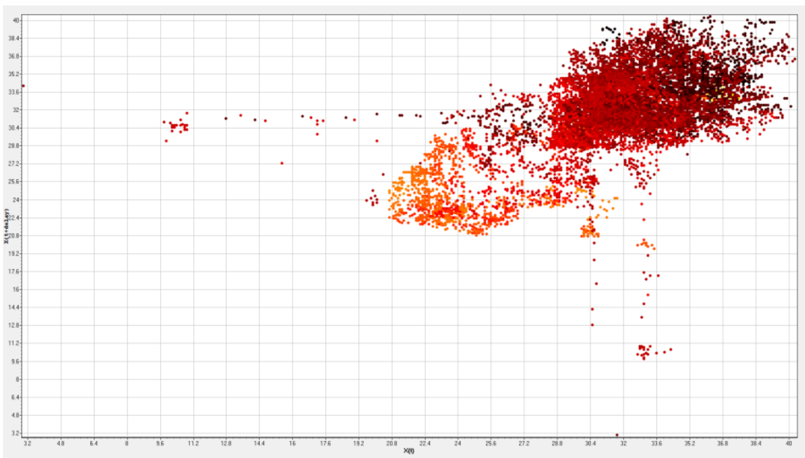
As with the Lorentz example, the Argonne grid is a deterministic system with specific characteristics.

Extracting characteristics of the dynamical behavior of a CI - The Argonne electrical grid as an example

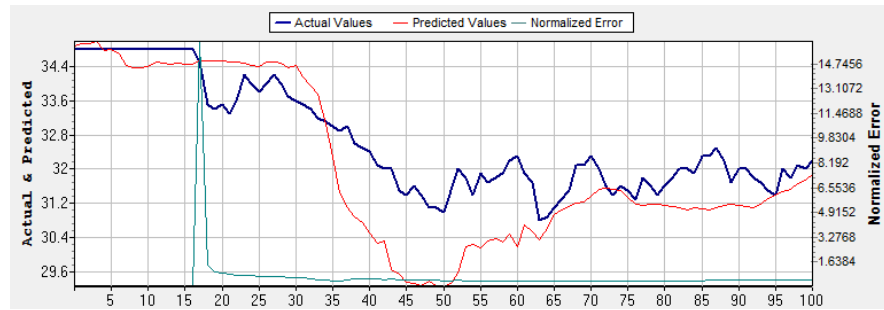
The correlation dimension D for the Argonne grid is ~ 3.5



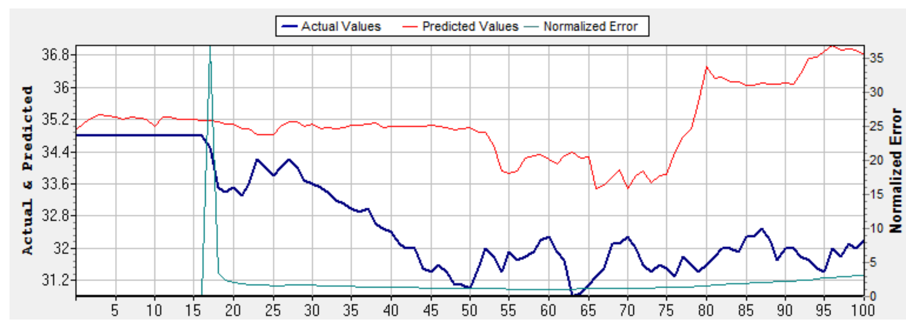
Short–term predictability of grid system evolution by NLD – The Argonne electrical grid as an example



Embedding dimension = 4



Embedding dimension = 5



- Argonne grid attractor may have a much higher dimension than 2 (~ 3-4?).
- Knowing the "correct" D is important.
- Affects short-term predictability.

Embedding in 4 dimensions improves short-term predictability.

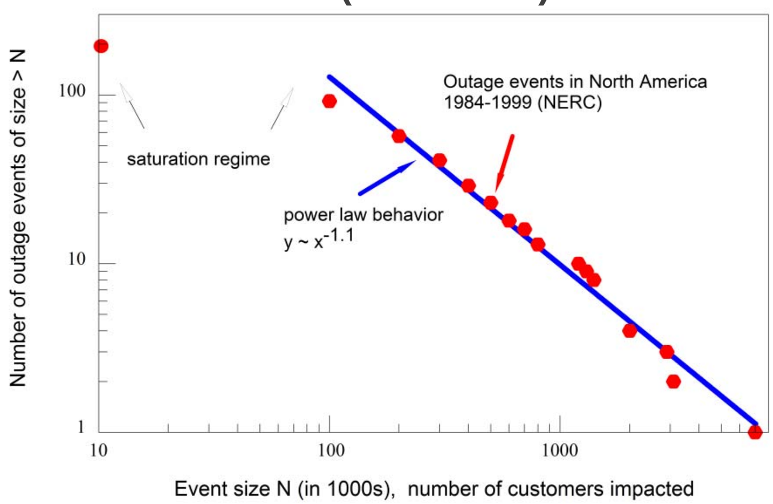
NLD-based methodology to monitor and detect the change in resiliency of a CI (1 of 2)

- Utilize available CI time-series data (historical and model-generated) for the CI of interest.
- Analyze and calculate all relevant constructs using NLD (e.g. MI, time delay, correlation dimension, predictability).
- Generate data base of correspondence between changes in these constructs and changes in the state (and resiliency) of the CI.

NLD-based methodology to monitor and detect the change in resiliency of a CI (2 of 2)

- Real-time update of the temporal evolution of these NLD-based constructs as more current CI data become available.
- Detection of the changes over time provide early ***quantitative*** warning of emerging ***qualitative*** changes in the CI behavior affecting the resiliency of the CI.
- Timely corrective action may then be taken.

North American outage statistics (1984–1999)



Early detection and timely corrective action may improve the resiliency of the CI of interest.

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